**Supplementary Materials**

**Unit: "Statistics"**

**Lesson: "Stratified Random Sampling"**

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| **Teaching Phases** | **GPT-4 Answer Examples** |
| Instructional Objective | (1) Students should understand and be able to clearly explain what "Stratified Random Sampling" is, and elaborate on its applications in data collection and research. Emphasize how to utilize stratified sampling more effectively to obtain samples in specific scenarios, enhancing data analysis skills and mathematical abstract thinking.  (2) Students should master the specific steps and methods of stratified sampling, and be able to design an implementation plan for stratified sampling independently. Recognize and identify potential problems related to stratified sampling that may be encountered in real life or theoretical research, improving logical reasoning and mathematical modeling proficiency.  (3) Students should be able to apply the knowledge of stratified sampling to analyze and solve some practical problems. For instance, in a survey study, how to design an effective sample using stratified sampling. They should not only be capable of application but also understand and explain the results, fostering intuitive imagination and mathematical computational skills. |
| Teaching Priorities and Challenges | **Teaching Priorities:** Understand and master the principles and applications of "Stratified Random Sampling", clarify its importance in data collection and research, and proficiently use logical reasoning and mathematical modeling skills to design a reasonable and effective stratified sampling scheme.  **Teaching Challenges:** Effectively applying "Stratified Random Sampling" when analyzing and solving real-world problems, including issues related to sample design and result interpretation. This requires a deep understanding and mastery of stratified random sampling knowledge, while also strengthening intuitive imagination and mathematical computation skills. |
| Pre-class Introduction | **[Case Introduction]:** Suppose we are a market research company commissioned by a large supermarket to conduct a study on consumer purchasing habits. Our task is to sample the supermarket's customers, but the supermarket's customers are numerous and diverse. We could simply randomly select some people, but this may result in underrepresentation of certain groups, such as young people or the elderly. How can we design a more effective sampling scheme?  **[Problem Chain 1]:** For such a complex population, what issues might arise if we only use simple random sampling? Is it possible that some groups may be under-sampled?  **[Problem Chain 2]:** How can we avoid this issue in simple random sampling? Can we classify the population based on certain characteristics and then draw a proportional sample from each category?  **[Problem Chain 3]:** Does this grouped sampling method have a specific name? What are its main steps and methods?  **[Teaching Activities and Design Intentions]:** By setting up a real-world scenario, students' interest in stratified random sampling is sparked. It allows students to ponder how to address practical challenges, naturally leading to the introduction of the concept of stratified random sampling. This approach enables students to better grasp the principles and applications of stratified random sampling, and they can effectively apply it to solve real-world problems in subsequent learning. |
| Explanation of New Knowledge | **Knowledge Point 1: What is stratified random sampling?**  **[Problem Chain 4]:** Based on the pre-class introduction, we have already categorized the supermarket's customer groups. Can such an operation solve the potential problems we discussed in simple random sampling?  **[Variant Question 1]:** Suppose we categorize based on two dimensions: gender and age, and randomly select a certain number of people from each category for the survey. Can this method better represent the overall customer group? What is this sampling method called?  **[Teaching Activities and Design Intentions]:** Interactive Q&A activity, encouraging students to think and answer questions, guiding them to discover the concept of stratified random sampling on their own. The goal of the instructional design is to let students understand through this activity that stratified random sampling is a more effective sampling method that can better represent the entire research group.  **Knowledge Point 2: How to carry out stratified random sampling?**  **[Problem Chain 5]:** We already understand the basic concept of stratified random sampling. So, how do we proceed with stratified random sampling? If we categorize the supermarket's customers by gender and age, how many people should we select in each category?  **[Variant Question 1]:** Suppose we categorize male customers by age groups: 18-25 years, 26-35 years, 36-45 years, and over 46 years. Female customers are also categorized in the same age brackets. How many people should we select in each category? How do we decide?  **[Teaching Activities and Design Intentions]:** Group discussion activity, encouraging students to understand and grasp the specific methods of stratified random sampling through discussion and exploration. Through this activity, students understand the factors and steps to be considered in stratified random sampling, enhancing their logical reasoning and mathematical modeling abilities.  **Knowledge Point 3: Advantages and disadvantages of stratified random sampling**  **[Problem Chain 6]:** Now that we know how to perform stratified random sampling, is this sampling method necessarily better than simple random sampling? If so, what are its advantages? If not, what are its disadvantages?  **[Variant Question 1]:** Suppose in our supermarket survey, we find that most shoppers are middle-aged females. In this case, which method is better, stratified random sampling or simple random sampling? Why?  **[Teaching Activities and Design Intentions]:** Classroom debate activity, allowing students to discuss and debate the pros and cons of stratified random sampling, enhancing their logical reasoning and data analysis abilities. Through this activity, students deeply understand the strengths and weaknesses of stratified random sampling and can better select and apply the appropriate sampling method to solve practical problems. |
| Consolidation and Improvement | **[Advanced Question]:** Suppose a supermarket has four main customer groups: students, office workers, retirees, and freelancers. Their consumption habits and needs in the supermarket might differ. Now, if the supermarket wants to understand the consumption needs and habits of each customer group, how should stratified sampling be carried out? And how should the number of samples to be drawn from each group be determined?  **[Answer Hint]:** First, stratify based on the supermarket's customer groups (students, office workers, retirees, and freelancers). Then, randomly draw a certain number of samples from each stratum. When determining the number of samples to be drawn from each stratum, consider the proportion of that stratum in the overall population, and factors like the importance of each group can also be considered.  **[Advanced Exercise 1]:** To gain a deeper understanding of customer shopping habits, the supermarket decided to add a question to the sample survey: What type of products do you usually purchase in the supermarket? Please design a simple survey form and explain your design rationale.  **[Answer Hint 1]:** When designing the survey form, it should include questions related to the customer's personal information (such as gender, age, occupation, etc.) as well as questions about shopping habits (like the most commonly purchased product types, shopping frequency, etc.). The design rationale is to understand the customer's shopping habits in as much detail as possible, providing the supermarket with a more effective basis for decision-making.  **[Advanced Exercise 2]:** If you were the statistician for this supermarket, how would you use the results of stratified random sampling to formulate more effective sales strategies? Please explain your rationale in as much detail as possible.  **[Answer Hint 2]:** The results of stratified random sampling can reveal the shopping needs and habits of various customer groups. This can assist the supermarket in formulating more accurate sales strategies targeting different groups. For instance, if survey results show that the student group prefers to buy snacks, while retirees prefer health foods, the supermarket could make corresponding adjustments in product arrangement, promotional activities, etc.  **[Teaching Activities and Design Intentions]:** In this section, through designing advanced questions and exercises, students are encouraged to apply the knowledge they've learned to real-world problems, enhancing their problem-solving and logical reasoning abilities. At the same time, these questions and exercises also offer students an opportunity to test and consolidate their knowledge. |
| Comprehensive Exercise | **[Integrated Question 1.1]:** Suppose a large city has 10 districts, each with a different population. Now the city government wishes to survey the quality of life of its residents and has decided to use the stratified random sampling method. Assuming the populations of the districts are A1, A2, ..., A10 respectively, if the city government decides to draw a total of 1,000 samples, how should it determine the number of samples drawn from each district?  **[Answer Hint 1.1]:** When performing stratified random sampling, the number of samples drawn from each stratum is usually proportional to its share of the overall population. Therefore, the number of samples drawn from each district should be the proportion of that district's population to the total population multiplied by the total number of samples. The specific formula is: Number of samples drawn from each district = (District's population / Total population) x Total number of samples.  **[Variant Exercise 1.2]:** If the city government decides to draw an equal number of samples from each district, how many samples should each district draw?  **[Answer Hint 1.2]:** If the city government decides to draw an equal number of samples from each district, then the number of samples each district should draw is the total number of samples divided by the number of districts. The specific formula is: Number of samples drawn from each district = Total number of samples / Number of districts.  **[Teaching Activity and Design Intent 1]:** Through specific examples and variant exercises, students are introduced to the operation method of stratified random sampling, understanding how to determine the number of samples drawn from each stratum in different situations. This also provides a practical application scenario to help students better understand and master the principles and methods of stratified random sampling.  **[Integrated Question 2.1]:** Suppose you are conducting a study on the physical health of middle school students nationwide and plan to randomly draw 1,000 middle school students as samples across the country. Considering differences in economy, culture, geography, etc., across the country, you decide to use the stratified random sampling method. How should you stratify? And determine the number of samples drawn from each stratum?  **[Answer Hint 2.1]:** Firstly, you can stratify based on geographic location (e.g., coastal eastern region, central region, western region), economic development level (e.g., first-tier cities, second-tier cities, third-tier cities, rural areas), or other important factors. Then, the number of samples drawn from each stratum can be determined based on its proportion in the total population. The specific formula is the same as in Question 1.1.  **[Variant Exercise 2.2]:** If you decide to draw an equal number of samples from each province, how many samples should you draw from each province?  **[Answer Hint 2.2]:** If you decide to draw an equal number of samples from each province, then the number of samples each province should draw is the total number of samples divided by the number of provinces. The specific formula is the same as in Question 1.2.  **[Teaching Activity and Design Intent 2]:** Through this question and variant exercise, students understand how to stratify when performing stratified random sampling and how to determine the number of samples drawn from each stratum based on the stratification. This also presents a practical research scenario, helping students recognize the application of stratified random sampling in real research. |
| Lesson Summary | **[Teacher Activity 1.1]:** Invite several students to share their understanding and feelings about what they have learned in today's class, including the concept of stratified random sampling, its implementation steps, and application scenarios. Encourage students to summarize and share in the form of mind maps.  **[Teacher Activity 1.2]:** Based on the students' sharing, the teacher will supplement and clarify, emphasizing the importance and application of stratified random sampling. The teacher can take this opportunity to answer questions students might have had during the learning process and provide feedback on students' strengths and areas for improvement demonstrated in the activity.  **[Summary of Knowledge from this Lesson]:**  (1) Stratified Random Sampling: Stratified random sampling is a sampling method where the population is divided into several non-overlapping sub-populations based on certain characteristics. Samples are then randomly selected from each sub-population (i.e., stratum), and all samples from the strata are combined as the final sample.  (2) Methods of Stratified Sampling: The main methods of stratified sampling are equal-size stratified sampling and proportional stratified sampling. Equal-size stratified sampling refers to drawing the same number of samples from each stratum, while proportional stratified sampling means that the number of samples drawn from each stratum is proportional to that stratum's size in the overall population.  (3) Application of Stratified Random Sampling: Stratified random sampling can better reflect the characteristics of the overall population, improving the efficiency and accuracy of sampling. As a result, it is widely used in many fields such as social surveys, market research, and educational evaluations.  (4) Operational Steps of Stratified Random Sampling: Determine the basis for stratification -> Divide into strata -> Decide the number of samples for each stratum -> Randomly sample within each stratum -> Combine the samples from all strata. |